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ANGLE OF TOTAL REFRACTION IN THE INTERACTION OF A PLANE SKEW SHOCK WAVE WITH THE INTERFACE OF TWO MEDIA

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The solution of practical problems associated with determination of the shock-wave configuration that results from the interaction of a stationary plane skew shock wave (SSW) with the interface of two media, is invariably related to determination of the critical angles of the interaction. The author suggests a method of analytical calculation of one of these angles within the framework of the applicability of the hydrodynamic theory of shock waves, namely, the angle of total refraction, denoted by φ_t *in some of the indicated works.*

Study of shock-wave configurations that result from break decay occurring in emergence of a stationary skew plane shock wave on an immovable plane interface between two media [1-4] has made it possible to establish that the form of the configuration and, consequently, the properties of the flow behind it depend completely on the relation between the angle of incidence and two characteristic angles: the angle φ_c of change of the pattern of the flow behind the skew shock wave from supersonic to subsonic and the angle φ_t of total refraction. The angle φ_c characterizes the material over which the initial skew shock wave moves, and in most cases it can be calculated analytically [1, 3]. It is more difficult to calculate φ_t, since it determines a two-wave break decay on the interface (Intf) of different-density media, when at the contact point two plane skew shock waves come in contact: the initial wave and the refracted wave (Fig. 1). To accomplish this calculation, it was usually necessary to solve a complicated transcendental equation [1-4]. We will show that φ_t can be found analytically proceeding from relations for the parameters of the flow behind the front of the skew shock wave that are general for the hydrodynamic theory of shock waves.

Let the space in the plane of the drawing (Fig. 1) be divided by the interface into two half-spaces; here the upper half-space is filled with a substance described by the equation of state $p = f(m)$, and the lower one, by the equation of state $p = g(n)$, where $m = \rho_{0,H}/\rho_H$ and $n = \rho_{0,L}/\rho_L$ ($n \le 1$, $m \le 1$). The velocity of the initial skew shock wave is D, while the interface is immovable. On realization of the regime of total refraction (Fig. 1) in a coordinate system tied to the contact point, the pressure jump behind the front of the skew shock wave and the refracted shock wave (RfrSW) can be written as follows:

$$p_H - p_{0,H} = \rho_{0,H} q^2 \sin^2 \varphi_t (1 - m) , \quad p_L - p_{0,L} = \rho_{0,L} q^2 \sin^2 \varphi_L (1 - n) , \tag{1}$$

where $p_{0,H} = p_{0,L} = p_0$ and $q = D/\sin \varphi_t$. The steadiness of the interaction is attributable to the fulfillment of two conditions: the pressure and the component of the flow velocity normal to the interface must coincide on both sides of this interface. The first steady-state condition and Eq. (1) give a relation between φ_L and φ_t:

$$\sin \varphi_L = \sin \varphi_t \sqrt{\left(\alpha \frac{1 - m}{1 - n}\right)}, \tag{2}$$

where $\alpha = \rho_{0,H}/\rho_{0,L}$. The quantities m and n should be considered to be known, since m can be determined from the relation

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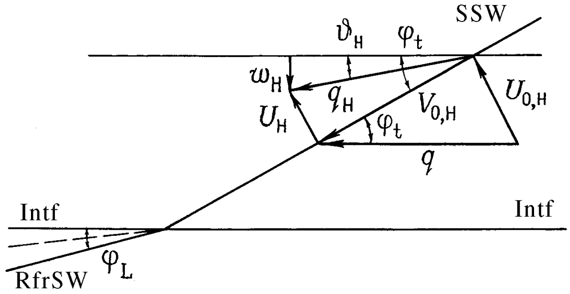

Fig. 1. Scheme of break decay in total refraction.

$$\frac{f(m)}{p_0} - 1 = \frac{\rho_{0,H} D^2}{p_0} (1 - m) , \tag{3}$$

while n can be determined from the equality $g(n) = f(m)$. In writting the second steady-state condition, we should seek an expression for the total velocity q_H and the angle of rotation of the flow ϑ behind the skew shock wave and the refracted shock wave. The continuity condition for the flow through the surface of the skew shock wave (Fig. 1) is

$$U_H = m U_{0,H} . \tag{4}$$

The flow component parallel to the front of the skew shock wave does not experience changes; this means that the total velocity of the flow behind the skew shock wave is

$$q_H = \sqrt{V_{0,H}^2 + U_H^2} = \sqrt{V_{0,H}^2 + m^2 U_{0,H}^2} = q \cos \varphi_t \sqrt{1 + m^2 \tan^2 \varphi_t} , \tag{5}$$

while the angle of rotation is

$$\vartheta_H = \varphi - \arccos \frac{V_{0,H}}{q_H} = \varphi - \arccos \sqrt{\left(\frac{1}{1 + m^2 \tan^2 \varphi_t} \right)} . \tag{6}$$

The component of the flow velocity normal to the interface is

$$w_H = q_H \sin \vartheta_H = q_H \sin \left(\varphi - \arccos \sqrt{\left(\frac{1}{1 + m^2 \tan^2 \varphi_t} \right)} \right) . \tag{7}$$

Having performed simple trigonometric transformations and having substituted the value of q_H from Eq. (5) into Eq. (7), we obtain

$$w_H = q (1 - m) \sin \varphi_t \cos \varphi_t . \tag{8}$$

After completely analogous operations carried out for the refracted shock wave we have

$$w_L = q (1 - n) \sin \varphi_L \cos \varphi_L \tag{9}$$

and using the second steady-state condition ($w_H = w_L$), we write

$$\frac{1 - m}{1 - n} \sin \varphi_t \cos \varphi_t = \sin \varphi_L \cos \varphi_L . \tag{10}$$

Simultaneous solution of Eqs. (2) and (10) makes it possible to obtain an exact formula for determining the angle of total refraction in the approximation of the hydrodynamic theory of shock waves:

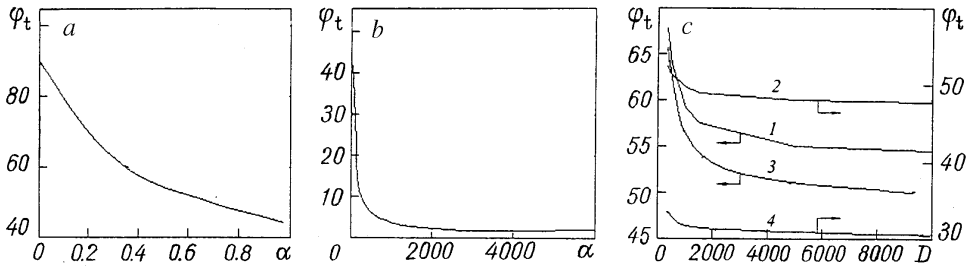

Fig. 2. Dependences $\varphi_t(\alpha)$ for $\alpha \le 1$ (a) and $\alpha > 1$ (b) and $\varphi_t(D)$ (c) for the systems air–krypton (1), krypton–air (2), air–CO_2 (3), and CO_2–air (4). All values of the angles are given in degrees.

$$\varphi_t = \arcsin \sqrt{\left(\frac{1 - \frac{1-n}{1-m}\alpha}{1-\alpha^2} \right)}. \tag{11}$$

Equation (11) confirms the conclusion [1-3] that φ_t is a parameter that characterizes a pair of materials (media) in contact on the interface under the given conditions of interaction (the parameters m, n, and α depend on D and are determined from the equations of state and the initial densities of the media).

Analyzing Eq. (11), we note that there are two domains of existence of φ_t, depending on α: 1) $\alpha \le 1$ and 2) $\alpha > 1$. In the first case, the break decay corresponds to [1-3], when the "upper" material is lighter than the "lower" (Fig. 1). Then, as $\alpha \to 0$, φ_t = arcsin (1) $\to \pi/2$; for a rigid surface, $\alpha = 0$ and $\varphi_t = \pi/2$. For materials close in characteristics, when $\alpha \to 1$ and m and n can be considered to be close in value ($(1-n)/(1-m) \approx 1$), $\varphi_t \to$ arcsin $(1/2^{0.5}) = \pi/4$. In the opposite case ($\alpha > 1$, the "upper" substance is heavier than the "lower" one), for close materials the result will be as previously, but for very large α we have $\varphi_t \to$ arcsin (0) = 0. A more specific form of the dependences $\varphi_t(\alpha)$ and $\varphi_t(D)$ for certain pairs of materials with different characteristics and aggregate state is given in Fig. 2.

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NOTATION

p, pressure; q, total flow velocity; U, component of the flow velocity perpendicular to the shock-wave front; V, component of the flow velocity parallel to the shock-wave front; w, component of the flow velocity perpendicular to the interface; D, velocity of the shock-wave front; m and n, ratio of the densities ahead of and behind the shock-wave front for the "upper" and "lower" medium, respectively; ρ, density; φ, angle between the corresponding shock wave and the interface; ϑ, angle of rotation of the flow behind the front of the corresponding skew shock wave; α, ratio of the initial densities of the "upper" and "lower" medium. Subscripts: t, angle of total refraction; H and L, upper and lower half-plane, respectively; 0, initial values.

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